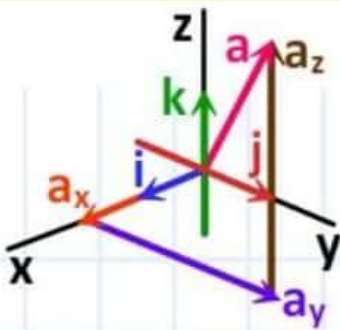


VECTOR

A quantity having both magnitude & direction



TYPES OF VECTORS

ZERO VECTOR

\vec{a} is zero vector
iff $|\vec{a}| = 0$
No Particular direction

UNIT VECTOR

\vec{a} is unit vector
iff $|\vec{a}| = 1$
Denote $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

EQUAL VECTORS

\vec{a} & \vec{b} are equal vectors iff
 $|\vec{a}| = |\vec{b}|$
& $\hat{a} = \hat{b}$

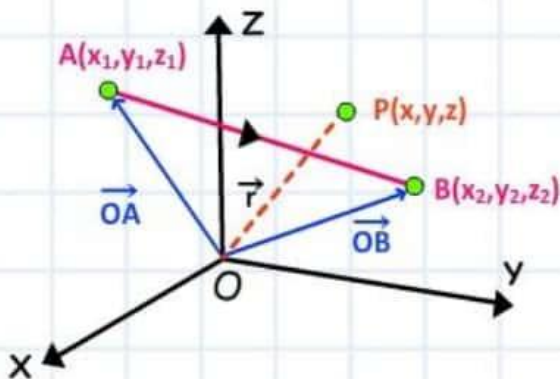
COLLINEAR VECTORS

\vec{a} & \vec{b} are collinear vectors
iff $\vec{a} \parallel \vec{b}$

COPLANER VECTORS

vector parallel to or lying on the same plane.

POSITION VECTOR AND SECTION FORMULA



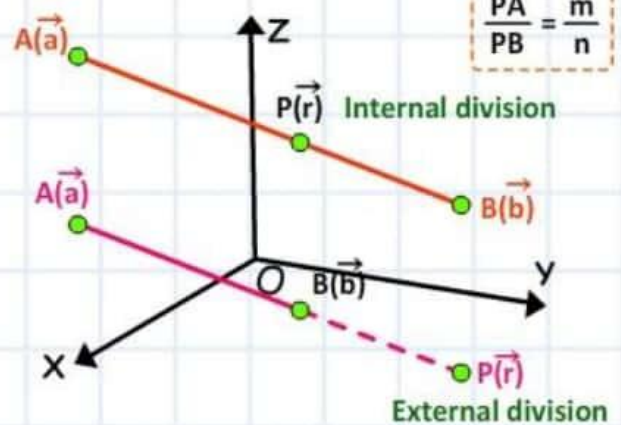
Position Vector

Position Vector of pt. P is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Vector joining two points A & B

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



Internal Section

$$\vec{r} = \frac{(n\vec{a}) + (m\vec{b})}{m+n}; m+n \neq 0$$

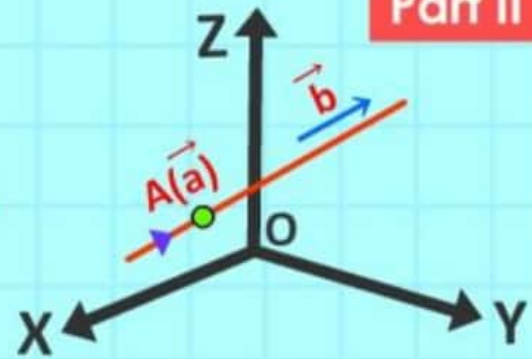
External Section

$$\vec{r} = \frac{(m\vec{b}) - (n\vec{a})}{m-n}; m-n \neq 0$$

VECTOR LINES

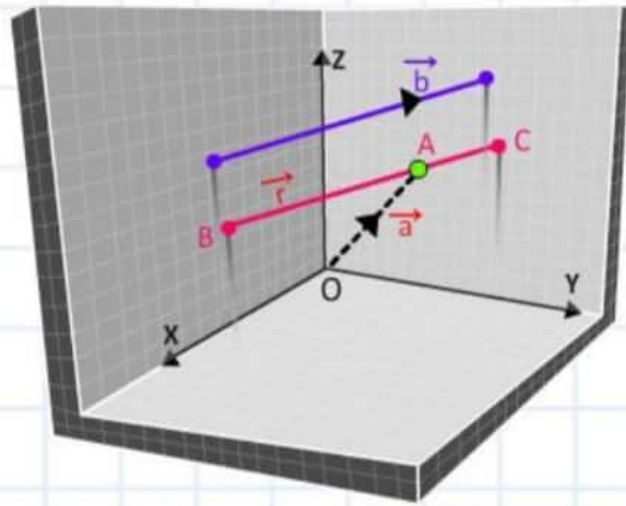
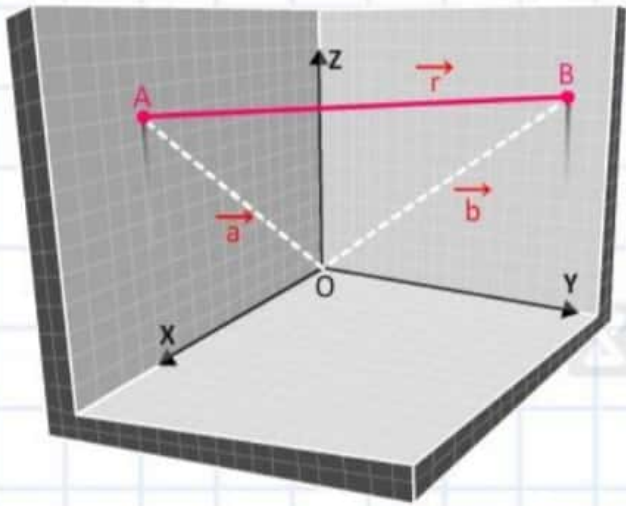
Lines : For a Vector equation of a line one needs

- A Point on line, let \vec{a} .
- A Parallel vector to the line, let \vec{b} .



When Two Points on Line are Known

Parallel to \vec{b} & Passes Through Point A



Equation of the Line AB

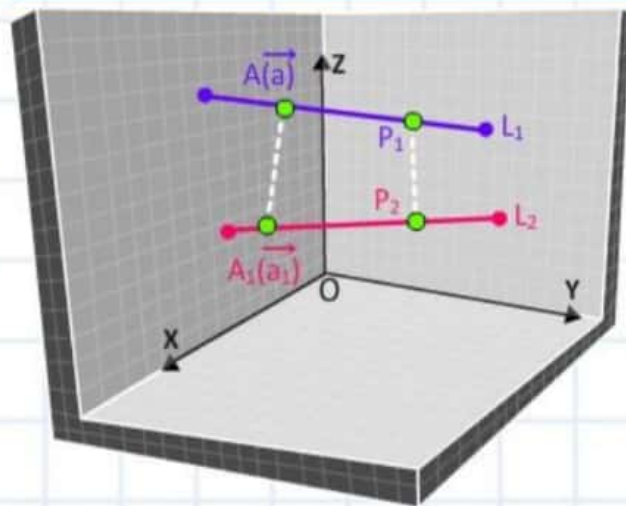
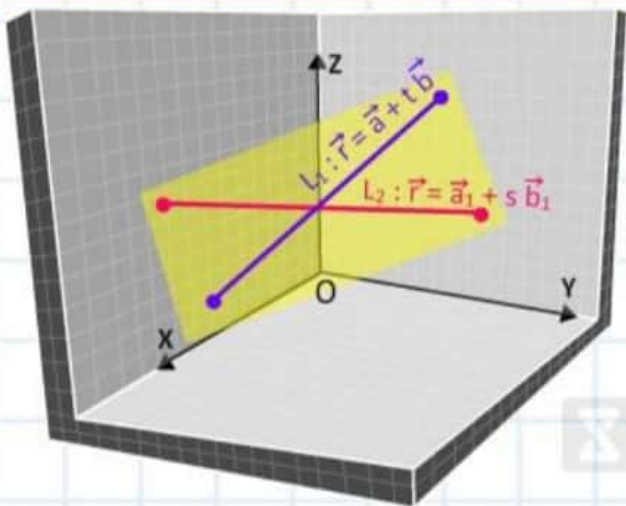
$$\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$$

Equation of the Line BC

$$\vec{r} = \vec{a} + t \vec{b}$$

Condition for Intersection of Two Straight Lines

Shortest distance between two Skew Lines



Condition of Intersection

- Line L_1 & L_2 will be coplaner
- Required Condition $[\vec{b} \vec{b}_1 (\vec{a} - \vec{a}_1)] = 0$

Shortest distance is measure along their common normal

$$\text{Shortest distance } P_1 P_2 = \frac{(\vec{b} \times \vec{b}_1) \cdot (\vec{a} - \vec{a}_1)}{|\vec{b} \times \vec{b}_1|}$$

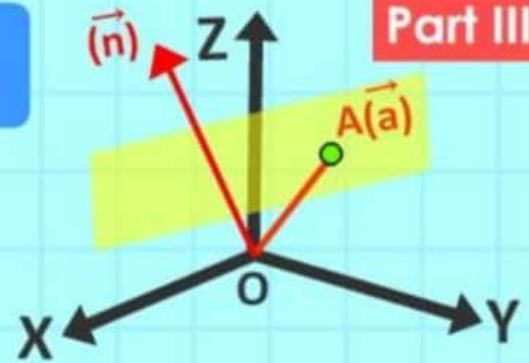
PLANE OF VECTORS

Part III

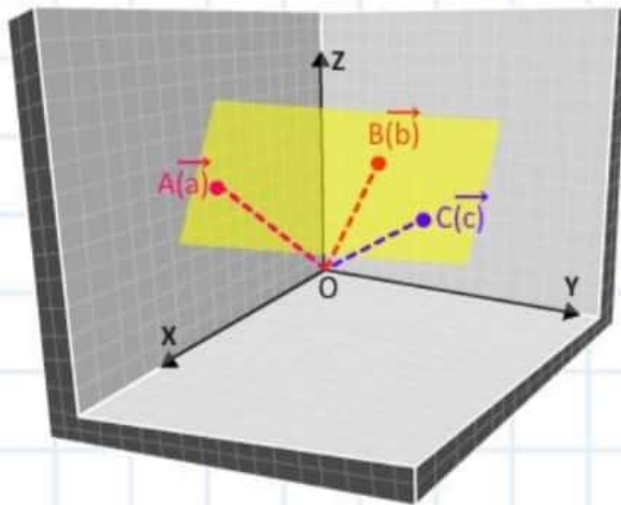
Planes : For Equation of a Plane, one needs

- ➔ A Point on Plane, let \vec{a} .
- ➔ A Perpendicular vector to the Plane, let \vec{n} .

Then Equation of Plane $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$



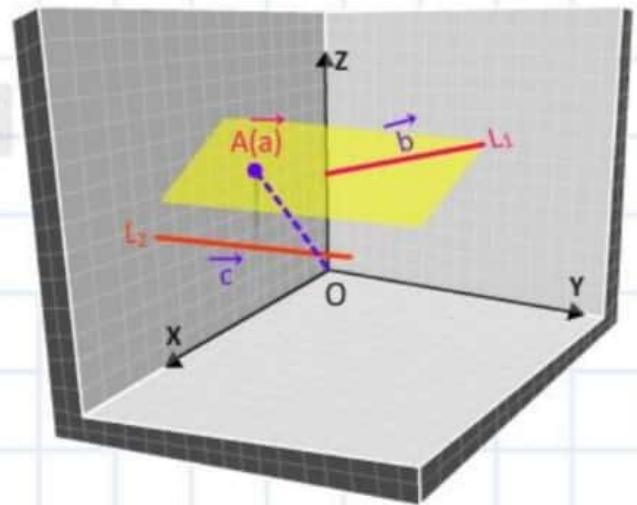
Plane Passing Through Three Points A,B,C



Equation of the Plane

$$(\vec{r} - \vec{a}) \cdot [(\vec{a} - \vec{b}) \times (\vec{b} - \vec{c})] = 0$$

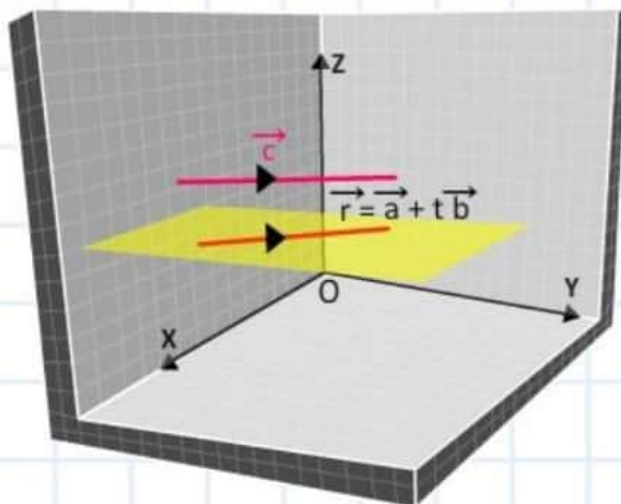
Plane Passing Through a Point and Parallel to two lines



Equation of the Plane

$$(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$$

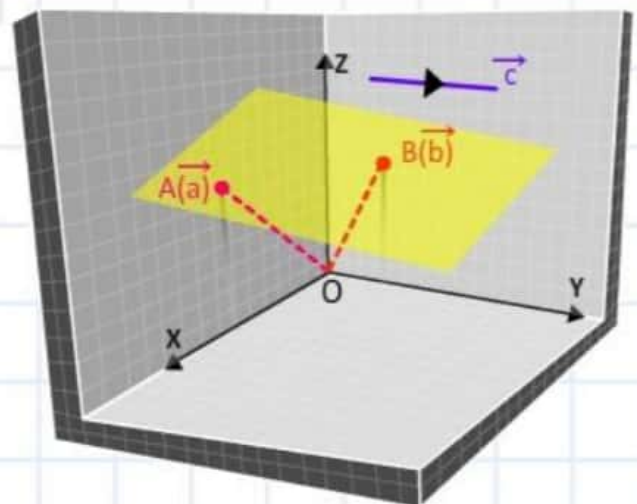
Plane Containing a Line and Parallel to another line



Equation of the Plane

$$\vec{r} \cdot (\vec{b} \times \vec{c}) = [\vec{a} \ \vec{b} \ \vec{c}]$$

Plane Passing Through Two Point and Parallel to a Lines



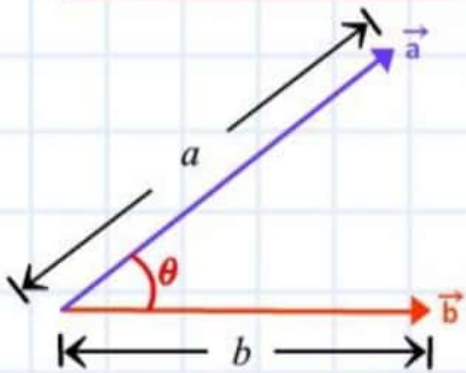
Equation of the Plane

$$\vec{r} \cdot [(\vec{b} - \vec{a}) \times \vec{c}] = [\vec{a} \ (\vec{b} - \vec{a}) \ \vec{c}] = [\vec{a} \ \vec{b} \ \vec{c}]$$

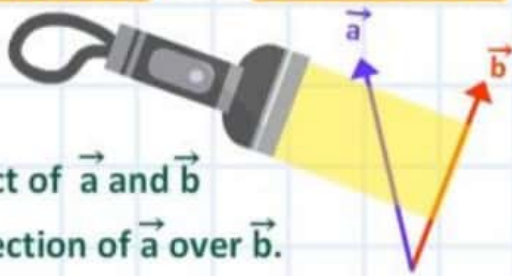


PRODUCT OF VECTORS

Scalar or Dot Product

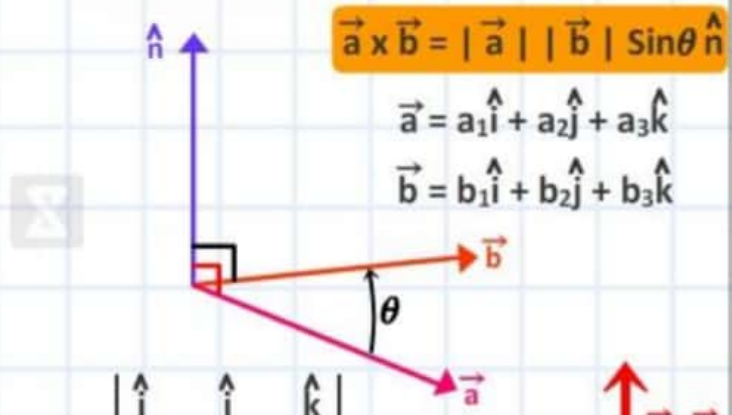


$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \quad \text{Or} \quad \vec{a} \cdot \vec{b} = a b \cos \theta$$



- Dot product of \vec{a} and \vec{b} is the projection of \vec{a} over \vec{b} .

Vector or Cross Product



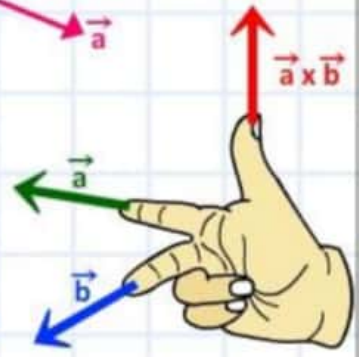
$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

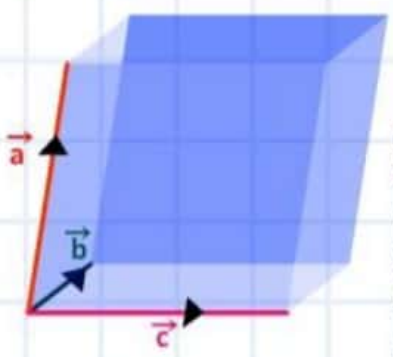
$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Direction of resultant vector can be found using **Right Hand Rule**



Scalar Triple Product



Scalar triple product represents the volume of a parallelepiped

SCALAR TRIPLE PRODUCT

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a} \ \vec{b} \ \vec{c}]$$

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

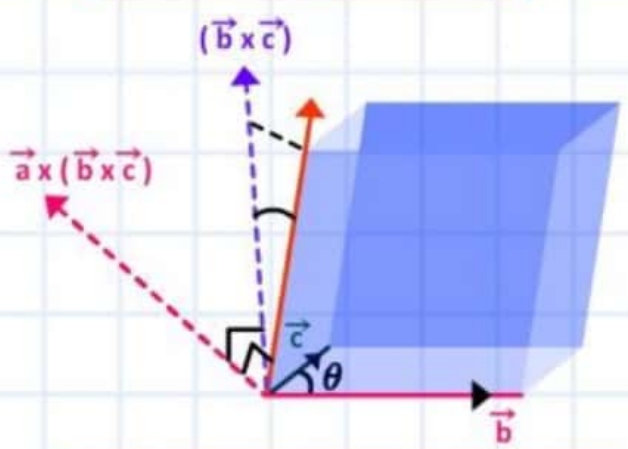
$$\vec{c} = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = [\vec{a} \ \vec{b} \ \vec{c}]$$

NOTE

- (i) \vec{a}, \vec{b} & \vec{c} are Coplanar iff $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

Vector Triple Product



$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$$

NOTE

- (i) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \vec{a} \cdot \vec{b} \times (\vec{c} \times \vec{d})$
 $= \vec{a} \cdot \{(\vec{b} \cdot \vec{d}) \vec{c} - (\vec{b} \cdot \vec{c}) \vec{d}\}$
 $= (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$
- (ii) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = [\vec{a} \ \vec{c} \ \vec{d}] \vec{b} - [\vec{b} \ \vec{c} \ \vec{d}] \vec{a}$
 Or $= [\vec{a} \ \vec{b} \ \vec{d}] \vec{c} - [\vec{a} \ \vec{b} \ \vec{c}] \vec{d}$